This document will attempt to mathematically justify why only two materials are necessary to create an optimized solution.

Since the band gap energy is a weighted average of the materials involved, it is mathematically evident (due to the way that averages work) that to obtain any goal band gap energy for a product, at least one material must have a band gap energy less than the goal and at least one material must have a band gap energy greater than the goal.

When only picking two materials to work with, we can show that the cheapest method of producing the goal band gap energy is by picking the most valuable material over and under the goal:

Where is the band gap energy, is the mass fraction, is the cost (the attribute to be minimized), and is the value (as defined in the tech brief). The subscript refers to the final product, refers to the material chosen over the goal and is the material under the goal.

It is self-evident that maximizing both values minimizes the cost. Maximizing reduces the term of the cost and similarly for .

Now we assert that given any method of constructing the product with materials, adding a non-zero mass fraction of a less valuable material will increase the cost of the product.

When constructing with materials (the summations are from to . represents material number and is the cost of the product of materials )

When constructing with materials (where is the new mass fraction necessary of the th material. The summation still goes from to and is the cost of the product of materials)

In order to prove our assertion, we need to prove that

Then, we can rewrite the inequality

Since the extra material is less valuable than the ones in use, , this inequality must hold as long as

Since all the mass fractions must sum to a whole, , it is evident that unless (in which case the extra material wasn’t actually used), it is cheaper to use fewer, but more valuable materials. Therefore, our model of using primarily two materials is the cheapest method of production.